7.4: Function Operations and Composition of Functions

[Algebra 2(Y)]

HCPS III

- ***Standard 9*:** Patterns, Functions, and Algebra: PATTERNS AND FUNCTIONAL RELATIONSHIPS: Understand various types of patterns and functional relationships.
- ***Benchmark MA.AII.9.4:*** Use the appropriate terminology and notation to define functions and their properties (e.g., domain, range, function composition, inverse, zeros).

Goal: Perform operations with functions, including composition of functions.

Operations on Functions

Operations on Functions

Let $f(x)$ and $g(x)$ be any two functions. You can add, subtract, multiply, or divide $f(x)$ and $g(x)$ to form a new function $h(x)$.

Operation	Definition	Example Let $f(x) = 2x$ and $g(x) = x + 1$.
Addition	$h(x) = f(x) + g(x)$	$h(x) = 2x + (x + 1) = 3x + 1$
Subtraction	$h(x) = f(x) - g(x)$	$h(x) = 2x - (x + 1) = x - 1$
Multiplication	$h(x) = f(x) \cdot g(x)$	$h(x) = (2x)(x + 1) = 2x^2 + 2x$
Division	$h(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$	$h(x) = \frac{2x}{x + 1}, x \neq -1$

The domain of h consists of the x-values that are in the domains of both f and g. When h involves division, the domain does not include x-values for which the denominator is equal to zero.

Example 1: Add and Subtract Functions

Let $f(x) = 3x^2$ and $g(x) = x - 1$. Find $h(x)$ and state its domain.

a.) $h(x) = f(x) + g(x)$ **b.)** $h(x) = f(x) - g(x)$

Extra Example 1: Add and Subtract Functions

Let $f(x) = 4x^2$ and $g(x) = x + 1$. Find $h(x)$ and state its domain.

a.) $h(x) = f(x) + g(x)$ **b.)** $h(x) = f(x) - g(x)$

Example 2: Multiply and Divide Functions

Let $f(x) = x^4$ and $g(x) = 3x$. Find $h(x)$ and state its domain.

a.) $h(x) = f(x) \circ g(x)$ **b.)** $h(x) = \frac{f(x)}{g(x)}$

Extra Example 2: Multiply and Divide Functions

Let $f(x) = x^3$ and $g(x) = 2x$. Find $h(x)$ and state its domain.

a.) $h(x) = f(x) \circ g(x)$

b.) $h(x) = \frac{f(x)}{g(x)}$

Composition of Functions

Composition of Functions: replacing the variable of a function with an entirely different function.

e.g., let $\boldsymbol{f(x) = 2x}$ and $\boldsymbol{g(x) = 5x + 3}$

The composition of $\boldsymbol{f}$ with $\boldsymbol{g}$ is:

$$\begin{aligned} \boldsymbol{f(g(x))} &= \boldsymbol{f(5x + 3)} \\ &= \boldsymbol{2(5x + 3)} \\ &= 10x + 6 \end{aligned}$$

The composition of $\boldsymbol{g}$ with $\boldsymbol{f}$ is:

$$\begin{aligned} \boldsymbol{g(f(x))} &= \boldsymbol{g(2x)} \\ &= \boldsymbol{5(2x) + 3} \\ &= 10x + 3 \end{aligned}$$

Example 3: Write a Composition of Functions

Let $f(x) = x^2$ and $g(x) = 3x - 1$. Find the following.

a.) $f(g(x))$ **b.)** $g\big(f(x)\big)$

c.) the domain of each composition.

Extra Example 3: Write a Composition of Functions

Let $f(x) = x^2$ and $g(x) = 2x + 3$. Find the following.

a.) $f(g(x))$ **b.)** $g\big(f(x)\big)$

c.) the domain of each composition.

Example 4: Evaluate a Composition of Functions

a.) Let $f(x) = x^2 - 4$ and $g(x) = 4x$. Evaluate $f(g(2))$.

b.) Let $f(x) = x^2 + 3$ and $g(x) = 5x$. Evaluate $f(g(2))$.

Example 5: Model a Real-World Situation

You have a coupon for \$15 off all Spring clothes over \$50. For this weekend only, the department store is offering an additional 20% off all purchases. Let x be the cost of your purchases. Then $f(x) = x - 15$ and $g(x) = 0.80x$. Find $g(f(x))$. Tell what it represents.