SEQUENCES AND SERIES

Note Title 17/10/2011

A sequence is a list of numbers generated by some rule

A series is the sum of a sequence (ie we add the terms)

Notation

The terms of a sequence are shown using subscripts:

u_1 means first term
u_{17} mean 17th term

A series is shown using sigma notation:

$$\sum_{i=1}^{i=7} u_i \quad \text{means} \quad u_1 + u_2 + u_3 + u_4 + u_5 + u_6 + u_7$$

A sequence can be generated by either a recurrence relation or an nth term rule.

Recurrence relation

This is a rule which generates a term from the previous terms

e.g ① $u_1 = 4$, $u_n = 2u_{n-1} - 3$ Find u_2, u_3, u_4, and u_5

$(n=2)$ $u_2 = 2u_1 - 3 = 5$
$(n=3)$ $u_3 = 2u_2 - 3 = 7$
$u_4 = 11$
$u_5 = 19$

② $u_1 = 1$, $u_2 = 1$, $u_n = u_{n-1} + u_{n-2}$ Find u_3, u_4, u_5, u_6

$(n=3)$ $u_3 = u_2 + u_1 = 2$
$(n=4)$ $u_4 = u_3 + u_2 = 3$
$u_5 = 5$
$u_6 = 8$ (Fibonacci sequence)

n^{th} term rule This is a rule which generates a term from the number of the term.

e.g. $u_n = 2^{n-1} + 3$ Find u_1, u_2, u_3, u_4, u_5 and u_{20}

$$\begin{aligned} u_1 &= 2^0 + 3 = 4 \\ u_2 &= 2^1 + 3 = 5 \\ u_3 &= 2^2 + 3 = 7 \\ u_4 &= 2^3 + 3 = 11 \\ u_5 &= 2^4 + 3 = 19 \\ &\vdots \\ u_{20} &= 2^{19} + 3 = 524291 \end{aligned}$$

(This is the same sequence as example ① above.
One advantage of an n^{th} term rule is that we can find any term without finding all the previous terms.)

Arithmetic Sequences and Series

An arithmetic sequence has the same difference (d) between each pair of terms.

ie/ its recurrence relation is $u_n = u_{n-1} + d$

We call the first term a. So the sequence is

u_1	u_2	u_3	u_4	.....	u_n
a ,	$a+d$,	$a+2d$,	$a+3d$		$a+(n-1)d$

So the n^{th} term rule is $\boxed{u_n = a + (n-1)d}$

To find the sum of an arithmetic series, we can do:-

$$S_n = a + (a+d) + (a+2d) + \cdots\cdots + a+(n-1)d$$

$$S_n = (a+(n-1)d) + (a+(n-2)d) + (a+(n-3)d) + \cdots + a$$

Add: $$2S_n = \underbrace{(2a+(n-1)d) + (2a+(n-1)d) + (2a+(n-1)d) + \cdots\cdots (2a+(n-1)d)}_{n \text{ terms}}$$

$$2S_n = n(2a + (n-1)d)$$

$$\boxed{S_n = \frac{n}{2}(2a + (n-1)d)}$$

Examples

(1) The first term of an arithmetic series is 3 and the 20th term is 79.

(a) Find the 30th term.

(b) Find the sum of the first 30 terms.

(a) $a = 3$

$$a + 19d = 79$$
$$19d = 76$$
$$d = 4$$

$$u_{30} = a + 29d$$
$$= 3 + 29 \times 4$$
$$= \underline{\underline{119}}$$

(b) $$S_{30} = \frac{30}{2}[6 + 29 \times 4]$$

$$= \underline{\underline{1830}}$$

122
× 15
1220
610
1830

p 85 Ex 6B Q 1 def, 2 cde, 7
p 87 Ex 6C Q 1 defg, 4, 5
p 90 Ex 6D Q 1 a-h, 2 a-d, 3, 4ac
p 92 Ex 6E Q 3, 5, 6
p 96 Ex 6F Q 1ac, 2ad, 3, 4, 7, 8, 10