QUADRATIC EXPRESSIONS AND EQUATIONS

Note Title 12/09/2011

Factorizing

① $3x^2 + 11x - 4$

$= \underbrace{3x^2 + 12x} \underbrace{- 1x - 4}$

$= 3x(x+4) - 1(x+4)$

$= (3x-1)(x+4)$

$3 \times -4 = -12$
Two numbers which
$\times$ to -12
and $+$ to 11
12 and -1

② $6x^2 - 11x - 10$

$= 6x^2 - 15x + 4x - 10$

$= 3x(2x-5) + 2(2x-5)$

$= (3x+2)(2x-5)$

$6 \times -10 = -60$
$\left.\begin{array}{l} \times \text{ to } -60 \\ + \text{ to } -11 \end{array}\right\}$ -15 and 4

Quadratic Equations

We can solve these by factorising (if possible), completing the square or using the quadratic formula.

Factorising

① $4x^2 = 12x$

$4x^2 - 12x = 0$ ← MUST make RHS $= 0$ first

$4x(x-3) = 0$

Either $4x = 0$ or $x - 3 = 0$

$\underline{x = 0}$ or $\underline{x = 3}$

② $6x^2 - 11x - 10 = 0$

$(3x+2)(2x-5) = 0$ (see above!)

Either $3x + 2 = 0$ or $2x - 5 = 0$

$\underline{x = -\frac{2}{3}}$ or $\underline{x = 2\frac{1}{2}}$

p 6 Ex 1E (all) — leave Q16 if stuck.
p 17 Ex 2B (1 - 17 odd)

Completing the Square

Examples

1. $x^2 + 10x + 28$

Halve the coefficient of x to find the number to put in the bracket.

$= x^2 + 10x + 25 + 3$

Make the constant term correct

Square the number found above.

$= (x + 5)^2 + 3$

2. $x^2 - 5x + 10$

Half of 5 is $2\frac{1}{2}$. $\left(2\frac{1}{2}\right)^2 = \left(\frac{5}{2}\right)^2$

$= \frac{25}{4} = 6\frac{1}{4}$

$= x^2 - 5x + 6\frac{1}{4} + 3\frac{3}{4}$

$= (x - 2\frac{1}{2})^2 + 3\frac{3}{4}$

3. $2x^2 - 12x + 7$

$= 2(x^2 - 6x + 3\frac{1}{2})$

$= 2(x^2 - 6x + 9 - 5\frac{1}{2})$

$= 2\left[(x - 3)^2 - 5\frac{1}{2}\right]$

$= 2(x - 3)^2 - 11$

Completing the square is useful for several purposes, one of which is solving quadratic equations.

e.g. ① $x^2 + 10x + 18 = 0$

$$x^2 + 10x + 25 - 7 = 0$$
$$(x+5)^2 - 7 = 0$$
$$(x+5)^2 = 7$$

($\sqrt{\ }$ both sides) $x + 5 = \pm\sqrt{7}$

$$\underline{x = \pm\sqrt{7} - 5}$$

p19 Ex 2D Q 1 - 7.

(p23 Ex 2G Q3)

② $ax^2 + bx + c = 0$

($\div a$) $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

(Half of $\frac{b}{a}$ is $\frac{b}{2a}$, $\left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$)

$$\underbrace{x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}} + \frac{c}{a} - \frac{b^2}{4a^2} = 0$$

$$\left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} - \frac{b^2}{4a^2} = 0$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$= \frac{b^2 - 4ac}{4a^2}$$

($\sqrt{\ }$ both sides) $x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$

$$= \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

($-\frac{b}{2a}$) $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

$$\underline{= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

which is the quadratic formula

The discriminant $b^2 - 4ac$

If $b^2 - 4ac < 0$ the equation has no real roots

If $b^2 - 4ac = 0$ the equation only has one (repeated) root, $\frac{-b}{2a}$.

If $b^2 - 4ac > 0$ the equation has two roots

('Root' is another word for solution.)

Examples

① The equation $x^2 + 2kx + k + 6 = 0$ has a repeated root. Find the possible values of k.

$$b^2 - 4ac = 0$$

NB brackets!

$$(2k)^2 - 4 \times 1 \times (k+6) = 0$$

$$4k^2 - 4(k+6) = 0$$

$$4k^2 - 4k - 24 = 0$$

$(\div 4)$

$$k^2 - k - 6 = 0$$

$$(k-3)(k+2) = 0$$

$$\underline{k = 3} \quad \text{or} \quad \underline{k = -2}$$

[If $k = 3$, the original eqn is $x^2 + 6x + 9 = 0$
$(x+3)(x+3) = 0$
$x = -3$ or -3

If $k = -2$, the original eqn is $x^2 - 4x + 4 = 0$
$(x-2)(x-2) = 0$
$x = 2$ or $x = 2$]

(2) For what values of c does the equation $x^2 + 8x + c$ have two distinct roots?

$$b^2 - 4ac > 0$$

$$64 - 4 \times 1 \times c > 0$$

$$64 - 4c > 0$$

$(\div 4)$

$$16 - c > 0$$

$$16 > c$$

$$\underline{\underline{c < 16}}$$

Green book p 21 Ex 2E Q 3, 5, 7, 9
(using formula)

p23 Ex 2F Q 2, 3
(using discriminant)

Black book p 46 Ex 3.3 Q 4a
p 49 Review 3 Q 6, 9.
(more discriminant)