LOGARITHMS
The essentials and the tactics

What is a Logarithm?

As with anything in mathematics, for one operation, there is an inverse operation for it. The Logarithm (Log) is the inverse function of the 10^x operation. The Log is another operation in a line of exponential functions, and the properties will be outlined later.

Basic Elements of a Logarithm

Let's establish the basics of the Logarithm function.

$$\text{Log}_{10}1000$$

The smaller number is the base of the logarithm. Many times the 10 is not written, as this is the default base for a logarithm. The 1000 is actually the answer of an exponent, as you will see below.

$\text{Log}_{10}1000 = \text{Log } 1000$

Sometimes, the base of the logarithm is not always a nice "10." There is a formula below to help in working with various bases. It may be a different number or it may be a variable. Note, however, that logarithm bases are never negative.

To properly refer to this function, we say:

"Log base 10 of 1000"

To solve this, look at the phrase and think of this:

"10 is the base and 1000 is the answer. What exponent satisfies $10^x = 1000$?"

Let's see what this phrase looks like as an equation:

$$\text{Log}_{10}1000 = x$$

Recall earlier that the operation 10^x is the inverse operation of the logarithm function, so we now have:

-> $10^{\text{Log}_{10}1000} = 10^x$

-> $1000 = 10^x$

-> $10^3 = 10^x$

Thus $x = 3$

From this calculation, let's draw some conclusions.

1.) This operation is reversible. If you are given $10^3 = 1000$, let 10 be the base of the logarithm and the answer be the exponent, so $\text{Log}_{10}1000 = 3$ is the opposite of the 10^x form.

2.) The general case for switching between 10^x and logarithm form is:

For integers a, b, and x in the form $a^x = b$, we write, in logarithm form:

$\text{Log}_a b = x$

This operation is reversible.

Operations with Logarithms

For some base b, and values m and n, we establish the following logarithm operations.

- Multiplication to Addition (Addition Rule)

  $Log_b(m*n) = Log_b m + Log_b n$

EX: $Log_{10}(5*x) = Log_{10}5 + Log_{10}x$
$= 0.698970004 + Log_{10}x$

- Division to Subtraction (Subtraction Rule)

$$Log_b\left(\frac{m}{n}\right) = Log_b m - Log_b n$$

EX: $Log_{10}\left(\frac{10}{x}\right) = Log_{10}10 - Log_{10}x$
$= 1 - Log_{10}x$

- Exponent Multiplier (Exponent Rule)

$$Log_b(m^n) = n*Log_b m$$

EX: $Log_{10}10^2 = 2*Log_{10}10$
$= 2*1$
$= 2$

(Compare this answer with $Log_{10}100$)

- Change of Base Formula (for bases other than 10)

$$Log_b m = \frac{Log_{10}m}{Log_{10}b}$$

EX: $Log_5 8 = \frac{Log_{10}8}{Log_{10}5}$
$= 1.292029674$

(Take $5^{1.292029674}$ and see that you get 8)

Now, let's try using these rules in a larger example.

EX: $Log_{10}\left(\frac{a^2b}{c^3}\right)^4 = 4Log_{10}\left(\frac{a^2b}{c^3}\right)$
$= 4(Log_{10}a^2b - Log_{10}c^3)$
$= 4(Log_{10}a^2 + Log_{10}b - Log_{10}c^3)$
$= 4(2Log_{10}a + Log_{10}b - 3Log_{10}c)$
$= 8Log_{10}a + 4Log_{10}b - 12Log_{10}c$

Final Notes

- A major point of logarithms is this: **DO NOT** confuse Log with Ln; this is the Natural Logarithm function, which is a completely different function from the standard logarithm.

Here's an example:

$Log_{10}100 = 2$

$Ln\ 100 = 4.605170186$

- NEVER use a negative number in a logarithm. There is no exponent that equals a negative number.

Example: Don't say $Log_{10}(-100)$ = NONE.

When rearranging it, we have $10^x = -100$, which is not possible.

- It is important to note that the logarithm of ANY base of 1 is 0 ($Log_b1 = 0$). This is simply because any number raised to the 0 exponent is 1.

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