JANUARY 2006

CXC

MATHEMATICS

Paper 02 - General Proficiency

2 hours 40 minutes

LIST OF FORMULAE

Volume of prism — $V = Ah$ where A is the area of a cross-section and h is the perpendicular length.

Volume of cylinder — $V = \pi r^2 h$ where r is the radius of the base and h is the perpendicular height.

Volume of a right pyramid — $V = \frac{1}{3}Ah$ where A is the area of the base and h is the perpendicular height.

Circumference — $C = 2\pi r$ where r is the radius of the circle.

Area of a circle — $A = \pi r^2$ where r is the radius of the circle

Area of trapezium — $A = \frac{1}{2}(a + b)h$ where a and b are the lengths of the parallel sides and h is the perpendicular height.

Roots of quadratic equations — If $ax^2 + bx + c = 0$,

then $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Trigonometry ratios

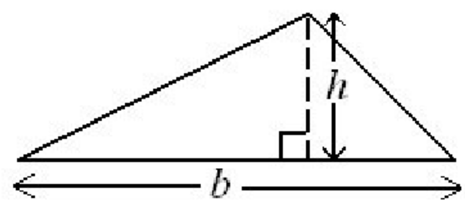
$$\sin \Theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos \Theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan \Theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

Area of triangle

Area of $\Delta = \frac{1}{2}bh$ where b is the length of the base and h is the perpendicular height.

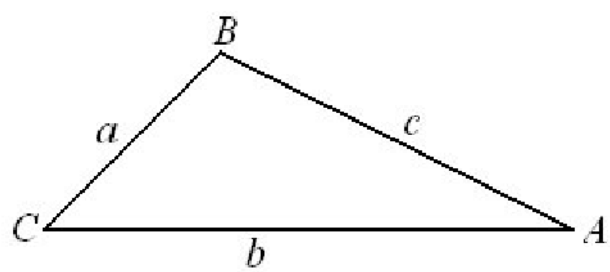


Area of $\Delta ABC = \frac{1}{2}ab \sin C$

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

Where $s = \frac{a+b+c}{2}$

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$



Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

SECTION I

Answer ALL the questions in this section.

All working must be clearly shown.

1. Using a calculator, or otherwise, calculate

(i) the exact value of $\dfrac{2\frac{1}{4} \times \frac{4}{5}}{\frac{3}{5} - \frac{1}{2}}$

(4 marks)

(ii) correct to 3 significant figures, the value of $18.75 - (2.11)^2$.

(3 marks)

(b) A loan of $12 000 was borrowed from a bank at 14% per annum.

Calculate

(i) the interest on the loan at the end of the first year **(2 marks)**

(ii) the total amount owing at the end of the first year. **(1 mark)**

A repayment of $7 800 was made at the start of the second year.

Calculate

(iii) the amount still outstanding at the start of the second year **(1 mark)**

(iv) the interest on the outstanding amount at the end of second year. **(1 mark)**

Total 12 marks

2. (a) Given that $m = -2$ and $n = 4$, calculate the value of $(2m + n)(2m - n)$.

(2 mark)

(b) Solve the simultaneous equations

$5x + 6y = 37$

$2x + 3y = 4$.

(4 marks)

(c) Factorise completely

(i) $4x^2 - 25$ **(2 marks)**

(ii) $6p - 9ps + 4q - 6qs$ **(2 marks)**

(iii) $3x^2 + 4x - 4$. **(2 marks)**

Total 12 marks

3. (a) Given the formula $s = \frac{1}{2}(u + v)t$, express u in terms of v, s, and t.

(3 marks)

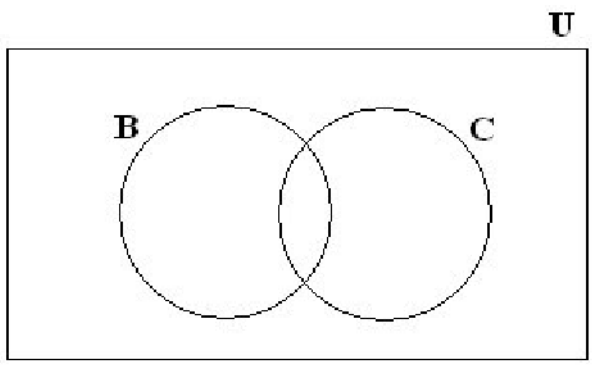
(b) On a certain day, 300 customers visited visited a bakery that sell bread and cakes.

70 customers bought cakes only

80 customers bought <u>neither</u> bread nor cakes

2x customers bought bread only

x customers bought both bread and cakes



(i) U represents the rest of customers visiting the bakery on that day, B represents the set of customers who bought bread, and C represents the set of customers who bought cake. Copy and complete the Venn diagram to illustrate the information.

(3 marks)

(ii) Write an expression in x to represent the TOTAL number of customers who visited the bakery on that day.

(2 marks)

(iii) Calculate the number of customers who bought bread ONLY.

(3 marks)

Total 11 marks

4. (a) The equation of line *l* is $y = 4x + 5$.

(i) State the gradient of any line that is parallel to *l*

(1 mark)

(ii) Determine the equation of the line parallel to *l* that passes through the point (2, -6).

(3 marks)

(b) The diagram below shows the positions of three cities, A, B, and C, on the north coast of Africa. The scale of the map is 1 : 20 000 000.

(Show all lines and angles used in your calculations in your answer sheet when answering the following questions)

(i) Measure and state, in centimetres, the length of line segment, BC.

(2 marks)

(ii) Hence, calculate in kilometres, the actual **shortest** distance from City B to City C.

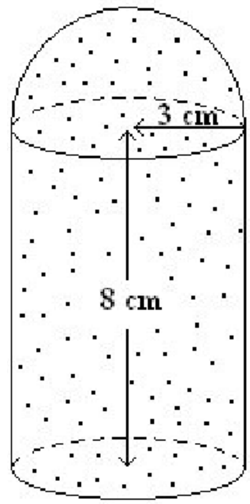
(2 marks)

(iii) Using a protractor, determine the bearing of B from A.

(4 marks)

Total 12 marks

5. The curved surface area of a cylinder = $2\pi rh$, where r is the radius and h is the height, and the surface area of a sphere is $4\pi r^2$.



The diagram above **not drawn to scale**, shows a **solid** glass paperweight which consists of a hemisphere mounted on a cylinder.

The radius of the hemisphere is 3 cm, the radius of the cylinder is 3 cm and its height is 8 cm.

(a) Calculate, using $\pi = 3.14$,

(i) the curved surface area of the cylinder

(2 marks)

(ii) the surface area of the hemisphere

(2 marks)

(iii) the TOTAL surface area of the **solid** paperweight.

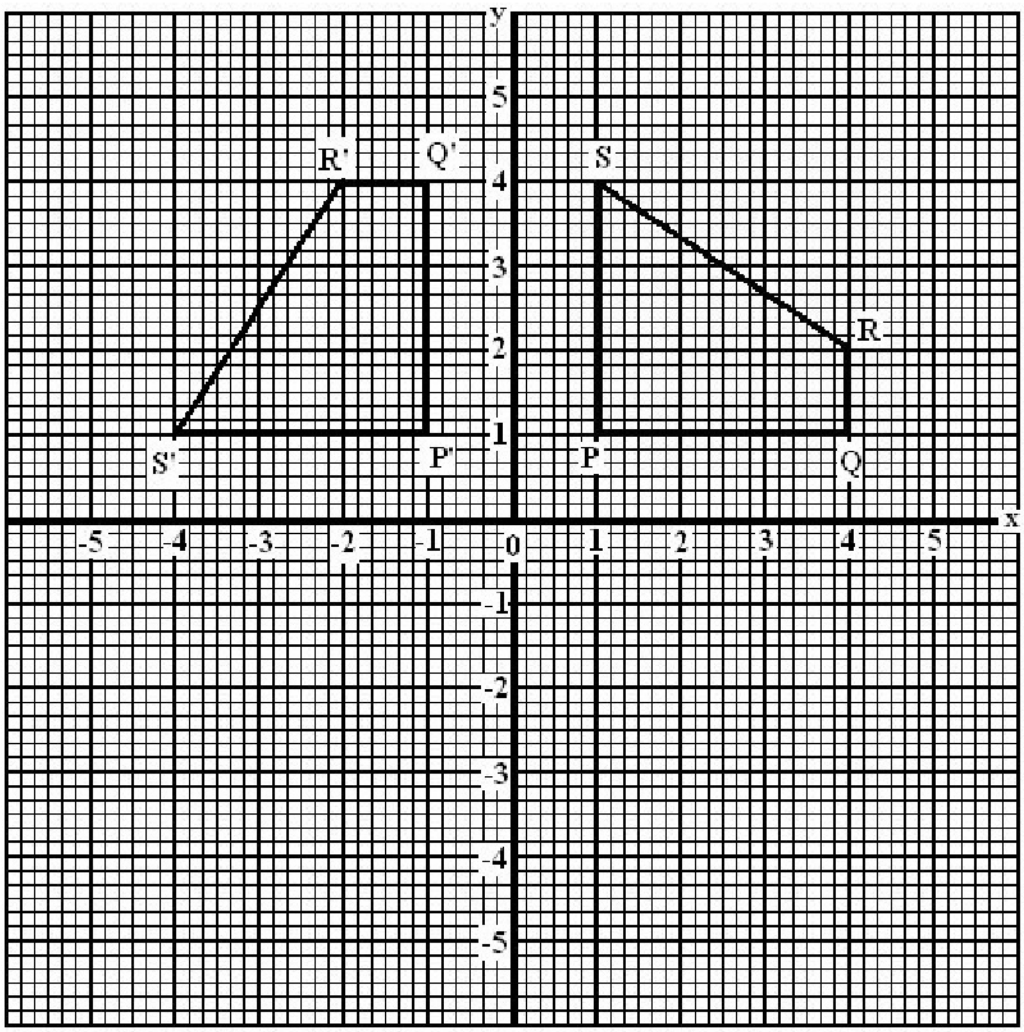
(2 marks)

(b) Using a ruler, a pencil, and a pair of compasses, construct the parallelogram KLMN, in which KL = 8cm, KN = 6 cm, and <LKN= 60°.

(5 marks)

Total 11 marks

6. The diagram below shows quadrilateral PQRS, and its image, quadrilateral P'Q'R'S', after it has been rotated.



(a) State the coordinates of the points R' and S'.

(2 marks)

(b) Describe the rotation completely.

(3 marks)

(c) On the answer sheet provided, draw and label quadrilateral P"Q"R"S" which is the image of P'Q'R'S' after it has been reflected in the x-axis.

(3 marks)

(d) Describe completely, the single transformation that maps quadrilateral PQRS onto P"Q"R"S".

(2 marks)

Total 10 marks

7. The data below are the lengths, to the nearest centimetre, of the right foot of the 25 students in a class.

14	18	20	22	24
15	18	20	22	25
16	18	21	22	25
16	19	22	23	26
17	19	22	23	27

(a) Copy and complete the following grouped frequency table for the data above.

Length of Right Foot (cm)	Frequency
14 - 16	4
17 - 19	__
20 - 22	8
______	5
26 - 28	2

(2 marks)

(b) State the lower boundary of the class interval 14 - 16.

(1 mark)

(c) State the width of the class interval 20 -22.

(1 mark)

(d) A student's right foot measured 16.8 cm. State the class interval in which this length would lie.

(1 mark)

(e) A student was chosen at random from the group, and the length of his right foot was measured. Calculate the probability that the length was GREATER THAN or EQUAL to 20 cm.

(2 marks)

(f) state the modal length of a student's right foot.

(1 mark)

(g) Calculate an estimate of the mean length of a student's right foot using the midpoints of the class intervals in (a) above.

(3 marks)

Total 11 marks

8. The path of a ball thrown in the air is given by the equation $h = 20t - 5t^2$ where h is the vertical distance above the ground (in metres) and t is the time (in seconds) after the ball was thrown.

The table below shows some values of t and the corresponding values of h, correct to 1 decimal place.

t	0	0.5	1	1.5	2	2.5	3	3.5	4
h	0.0	8.8	15	18.8		18.8		8.8	0.0

(a) Copy and completer the table of values.

(2 marks)

(b) Draw the graph of $h = 20t - 5t^2$ for $0 \leq t \leq 4$. You may proceed as follows:

(i) Using a scale of 2 cam to represent 0.5 seconds, draw the horizontal t-axis for $0 \leq t \leq 4$.

(1 mark)

(ii) Using a scale of 1 cm to represent 1 metre, draw the vertical h-axis for $0.0 \leq h \leq 21.0$.

(1 mark)

(iii) Plot the points from your table of values on the axis drawn.

(2 marks)

(iv) Join the points with a smooth curve.

(1 mark)

(c) Using your graph, Calculate estimates of

(i) the GREATEST height above the ground reached by the ball

(1 mark)

(ii) the number of seconds for which the ball was MORE than 12 metres above the ground

(2 marks)

(iii) the interval of time during which the ball was moving upwards.

(1 mark)

Total 11 marks

SECTION II

Answer TWO questions in this section

ALGEBRA AND RELATIONS, FUNCTIONS AND GRAPHS

9. (a) Solve the pair of simultaneous equations

$2x + y = 7$

$x^2 - xy = 6.$

(b) Express $4x^2 - 12x - 3$ in the form $a(x + h)^2 + k$, where a, h and k are real numbers.

(3 marks)

(c) Using your answer from (b) above, or otherwise, calculate

(i) the minimum value of $4x^2 - 12x - 3 = 0$ expressing your answer to 3 significant figures.

(4 marks)

Total 15 marks

10. (a) A shop stocks x Sonix and y Zent radios. It has shell space for up to 20 radios.

(i) Write an inequality to represent this information

(1 mark)

The owner of the shop spends \$150 to purchase each Sonix radio and \$300 for each Zent radio, she has \$4 500 to spend on the purchase of these radios.

(ii) Write an inequality to represent this information.

(1 mark)

The owner of the shop decides to stock at least 6 Sonix and at least 6 Zent radios.

(iii) Write TWO inequalities to represent this information.

(2 marks)

(b) (i) Using a scale of 2 cm to represent 5 Sonix radios and 2 cm to represent 5 Zent radios, draw the horizontal axis for $0 \leq y \leq 25$.

(1 mark)

(ii) On these axes, draw the four boundary lines for the four inequaliteies written in (a) (i), (ii) and (iii) above.

(4 marks)

(iii) Shade the region on your graph that satisfies ALL four of the inequalities written in (a) (i), (ii) and (iii) above.

(1 mark)

(iv) State the coordinates of the vertices of the shaded region.

(2 marks)

(c) The owner of the shop sells the radios to make a profit of \$80 on each Sonix and \$100 on each Zent radio.

(i) Express the TOTAL profit in terms of x and y.

(1 mark)

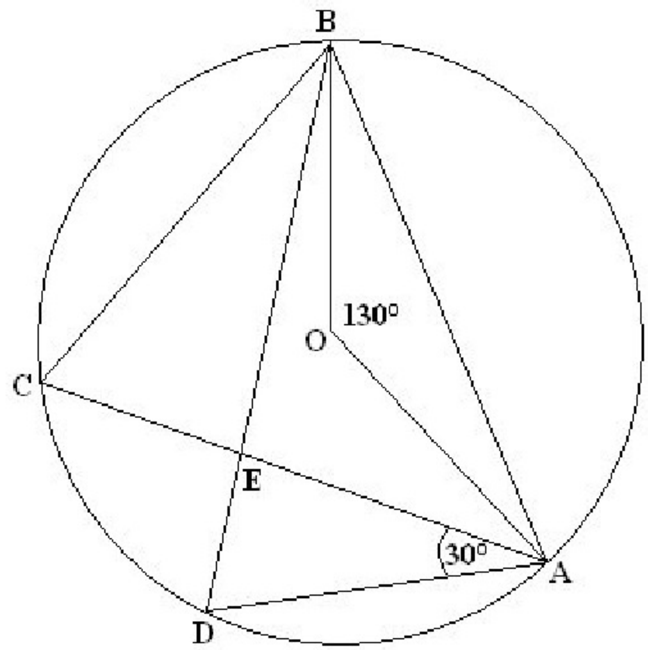
(ii) Calculate the MAXIMUM profit.

(2 marks)

Total 15 marks

11.

GEOMETRY AND TRIGONOMETRY



In the diagram above, **not drawn to scale**, o is the centre of the circle, <AOB = 130°, <DAE = 30°, and AEC and BED are chords of the circle.

(a) Calculate the size of EACH of the following angles, giving reasons for EACH step of your answers.

(i) <ACB **(2 marks)**

(ii) <CBD **(2 marks)**

(iii) <AED **(2 marks)**

(b) Show that ΔBCE and ΔADE are similar.

(3 marks)

(c) Given that CE = 6 cm, EA = 9.1 cm and DE = 5 cm,

(i) calculate the length of EB

(3 marks)

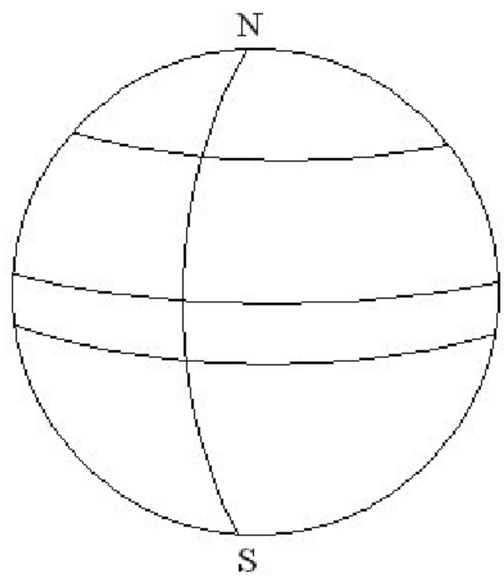
(ii) calculate correct to 1 decimal place the area of ΔAED.

(3 marks)

Total 15 marks

12. In this question assume that the earth is a sphere of radius 6 370 km.

The diagram below shows a sketch of the earth with the north pole, N, and the south pole, S, labelled.



The four arcs on the diagram represent the equator, the Greenwich Meridian, latitude 6°N and latitude 52°N.

(a) Sketch the diagram and label the

(i) equator

(ii) Greenwich Meridian

(iii) latitude 6°N

(6 marks)

(b) The Greenwich Meridian passes through London (52°N, 0°) and Accra (6°N, 0°).

(i) Show on your diagram the position, L, of London and A, of Accra.

(2 marks)

(ii) Calculate, to the NEAREST kilometre, the **shortest** distance between London and Accra along their common circle of longitude. Use $\pi = 3.14$.

(4 marks)

(c) Tropical Storm Kyle was reported to be located 5 470 km due west of Accra.

(i) Show on your diagram a possible point of location of Kyle, K.

(1 mark)

(ii) Calculate the radius of the circle of latitude on which k lies.

(2 marks)

Total 15 marks

VECTORS AND MATRICES

13. The points A(1, 2) B(5, 2) C(6, 4) and D(2, 4) are the vertices of the quadrilateral ABCD.

(a) Express in the form $\begin{pmatrix} x \\ y \end{pmatrix}$

(i) The position vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$, and $\overrightarrow{OD}$ where O is the origin (0, 0)

(2 marks)

(ii) the vectors $\overrightarrow{AB}$ and $\overrightarrow{DC}$.

(2 marks)

(b) Calculate $\left|\overrightarrow{AB}\right|$ and hence determine the unit vector in the direction of $\overrightarrow{AB}$.

(2 marks)

(c) Using the answers in (a) (ii),

(i) state TWO geometrical relationships between the line segments AB and DC

(2 marks)

(ii) explain why ABCD is a parallelogram.

(2 marks)

(d) Using a vector method, determine the position vector of G, the midpoint of the line AC.

Hence state the coordinates of the point of intersection of the diagonals AC and BD of parallelogram ABCD.

(5 marks)

Total 15 marks

14. (a) The matrix $L = \begin{pmatrix} x & 4 \\ 1 & x \end{pmatrix}$.

(i) Calculate, in terms of x, the determinant of L.

(ii) Calculate the values of x given that L is singular.

(3 marks)

(b) The matrix $M = \begin{pmatrix} 3 & 1 \\ 2 & 6 \end{pmatrix}$.

(i) Find M^{-1}, the inverse of M.

(ii) Hence, calculate the value of x and y for which $M\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12 \\ -8 \end{pmatrix}$.

(6 marks)

(c) The image, (x', y'), of any point, (x, y), under a transformation N is given by the equation

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

Calculate

(i) the image, (x' , y'), of (3, -1) under N

(3 marks)

(ii) the coordinates of the point, (x, y), which is mapped by N onto (7, 4).

(3 marks)

Total 15 marks

END OF TEST