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- I. Model Problems.
- II. Practice
- III. Challenge Problems
- IV. Answer Key

Web Resources

Inverse Functions

www.mathwarehouse.com/algebra/relation/inverse-of-function.php

Functions and Relations

www.mathwarehouse.com/algebra/relation/

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Inverse Functions

The function $g(x)$ are inverses of each other $f(x)$ if $g(f(x)) = x$ and $f(g(x)) = x$.

The inverse of the function $f(x)$ is indicated with the notation $f^{-1}(x)$, read f inverse (this notation does **not** mean $\frac{1}{f(x)}$).

I. Model Problems

In this example we will find the inverse of a discrete function for a given as a list of ordered pairs.

Example 1: If $f = \{(3, 2), (4, -6), (-2, 11), (5, 5)\}$ find $f^{-1}(x)$.

When finding the inverse exchange x and y . The ordered pairs (x, y) become (y, x) .

Answer: $f^{-1} = \{(2, 3), (-6, 4), (11, -2), (5, 5)\}$

In these examples we will find the inverse of functions given as an equation.

Example 2: If $f(x) = 3x + 10$ find $f^{-1}(x)$.

Write function in terms of y .

$$\begin{aligned} f(x) &= 3x + 10 \\ y &= 3x + 10 \end{aligned}$$

When finding the inverse exchange x and y .

Solve for y .

$$\begin{aligned} x &= 3y + 10 \\ -10 & \quad -10 \\ \frac{x - 10}{3} &= \frac{3y}{3} \\ \frac{x - 10}{3} &= y \end{aligned}$$

Rewrite as $f^{-1}(x)$.

$$f^{-1}(x) = \frac{x - 10}{3}$$

Answer: $f^{-1}(x) = \frac{x - 10}{3}$

Example 3: If $f(x) = \sqrt{x + 12}$ find $f^{-1}(x)$.

Write function in terms of y .

$$\begin{aligned} f(x) &= \sqrt{x + 12} \\ y &= \sqrt{x + 12} \\ x &= \sqrt{y + 12} \end{aligned}$$

When finding the inverse exchange x and y .

Solve for y . Square both sides of the equation.

$$\begin{aligned} x^2 &= (\sqrt{y + 12})^2 \\ x^2 &= y + 12 \\ -12 & \quad -12 \\ x^2 - 12 &= y \\ f^{-1}(x) &= x^2 - 12 \end{aligned}$$

Rewrite as $f^{-1}(x)$.

Answer: $f^{-1}(x) = x^2 - 12$

II. Practice Problems

Solve.

1. Is $g(x) = \frac{1}{2}x - 2$ the inverse of $f(x) = 2x + 4$? Justify your answer.
2. Is $g(x) = 4x + 24$ the inverse of $f(x) = \frac{1}{4}x + 6$? Justify your answer.
3. Is $h(x) = x^2 - 2$ the inverse of $g(x) = \sqrt{x + 2}$? Justify your answer.
4. Is $h(x) = x^2$ the inverse of $g(x) = \sqrt{x}$? Justify your answer.

Find the inverse of the given function.

5. $f = \{(1,3), (2,-5), (3,6)\}$
6. $g = \{(-4,1), (-3,2), (0,0), (1,10)\}$
7. $h = \{(-1,-1), (0,0), (3,3), (6,6)\}$
- 8.

| x | y |
|-----|-----|
| -3 | -2 |
| -1 | 2 |
| 0 | 4 |
| 1 | 6 |
| 3 | 8 |

- 9.
10. $f(x) = 3x - 7$

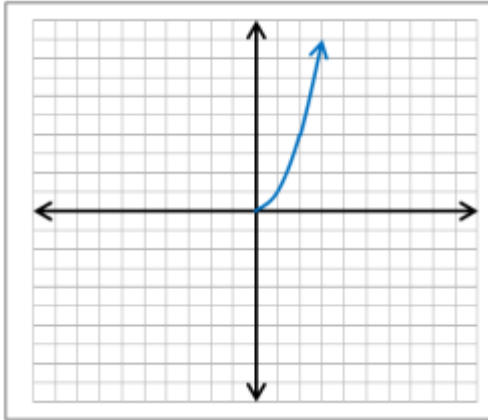
| x | y |
|-----|-----|
| -3 | 0 |
| 1 | 2 |
| 6 | 3 |
| 13 | 4 |
| 22 | 5 |

11. $g(x) = -4x + 5$
12. $h(x) = \frac{2}{5}x + 6$
13. $f(x) = \frac{3x+4}{7}$
14. $g(x) = \frac{1}{4}x + 6$
15. $g(x) = -3x - 10$
16. $f(x) = \sqrt{x - 4}$
17. $g(x) = \sqrt{2x + 8}$
18. $h(x) = \sqrt{3x} - 6$
19. $f(x) = 4\sqrt{x}$
20. Graph the inverse of $f(x) = 4x - 12$.

Challenge Problems

1. Graph the inverse of $f(x) = \sqrt{x+1}$ (Hint: identify the domain of $f(x)$).

2. Graph the inverse of the function graphed below.



scale of x and y-axes is 1.

3. Find the inverse of the function $f(x) = \sqrt[3]{x+4}$.

4. Find the error in the student's work for the following problem:

If $f(x) = \frac{x-7}{x}$, find $f^{-1}(x)$.

Given $f(x) = \frac{x-7}{x}$

Step 1 $y = \frac{x}{x-7}$

Step 2 $x = \frac{x}{y-7}$

Step 3 $x(x) = \left(\frac{y-7}{x}\right)x$

Step 4 $x^2 = \frac{y-7}{x}$

Step 5 $x^2 + 7 = y$

Step 6 $f^{-1}(x) = x^2 + 7$

5. Find the inverse of the function $f(x) = \frac{x-2}{x}$.

IV. Answer Key

1. yes; $g(f(x)) = g(2x + 4) = \frac{1}{2}(2x + 4) - 2 = x + 2 - 2 = x$;

$$f(g(x)) = g\left(\frac{1}{2}x - 2\right) = 2\left(\frac{1}{2}x - 2\right) + 4 = x - 4 + 4 = x$$

2. no; $g(f(x)) = g\left(\frac{1}{4}x + 6\right) = 4\left(\frac{1}{4}x + 6\right) + 24 = x + 24 + 24 = x + 48 \neq x$

3. yes; $h(g(x)) = h(\sqrt{x+2}) = (\sqrt{x+2})^2 - 2 = x + 2 - 2 = x$;

$$g(h(x)) = g(x^2 - 2) = \sqrt{(x^2 - 2) + 2} = \sqrt{x^2} = x$$

4. yes; $h(g(x)) = h(\sqrt{x}) = (\sqrt{x})^2 = x$; $g(h(x)) = g(x^2) = \sqrt{x^2} = x$

5. $f^{-1} = \{(3,1), (-5,2), (6,3)\}$

6. $g^{-1} = \{(1,-4), (2,-3), (0,0), (10,1)\}$

7. $h^{-1} = \{(-1,-1), (0,0), (3,3), (6,6)\}$

8.

| x | y |
|----|----|
| -2 | -3 |
| 2 | -1 |
| 4 | 0 |
| 6 | 1 |
| 8 | 3 |

9.

| x | y |
|---|----|
| 0 | -3 |
| 2 | 1 |
| 3 | 6 |
| 4 | 13 |
| 5 | 22 |

10. $f^{-1}(x) = \frac{x+7}{3}$

11. $g^{-1}(x) = \frac{x-5}{4}$

12. $h^{-1}(x) = \frac{5}{2}x - 15$

13. $f^{-1}(x) = \frac{7x-4}{3}$

14. $g^{-1}(x) = 4x + 24$

15. $g^{-1}(x) = \frac{x+10}{3}$

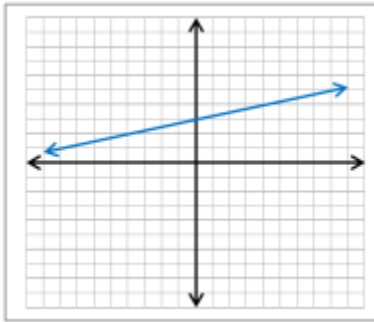
16. $f^{-1}(x) = x^2 - 4$

17. $g^{-1}(x) = \frac{x^2-8}{2}$

18. $h^{-1}(x) = \frac{x^2+12x+36}{3}$

19. $f(x) = \frac{x^2}{16}$

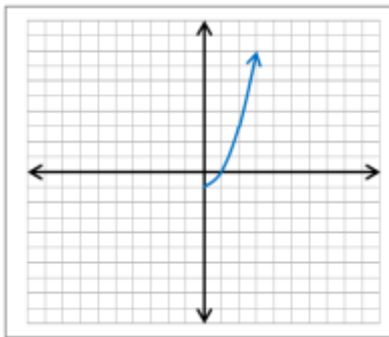
20.



scale of x and y-axes is 1.

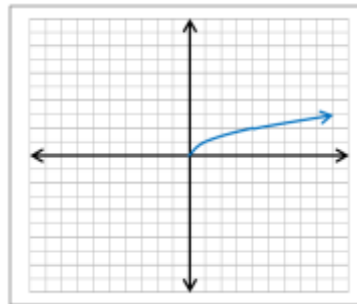
Challenge Problems

1.



scale of x and y-axes is 1.

2.



scale of x and y-axes is 1.

3. $g^{-1}(x) = x^3 - 4$

4. Step 2; replace both x with y

5. $f^{-1}(x) = \frac{-2}{x-1}$