e.g. 1 : The function $f(x)=\frac{2}{x}$ is called a rational function.

The graph of $f(x)$ \{discussed in the section on LIMITS\} has the x -axis and y -axis as asymptotes.
On different axes draw the graphs of
(i) $y=\frac{2}{x-1}$,
(ii) $y=\frac{x+2}{x}$,
(iii) $y=\frac{x+1}{x-1}$
answer :
(i) $\frac{2}{x-1}=f(x-1)$

This represents a translation through 1 unit in the x-direction. The asymptotes are the $y$-axis and $x=1 \quad\{$ dotted $\}$
(ii)

$$
\frac{x+2}{x}=\frac{2}{x}+1=f(x)+1
$$

This represents a translation through 1 unit in the $y$-direction. The asymptotes are now the x -axis and $\mathrm{y}=1 \quad\{$ dotted \}
(iii) $\frac{x+1}{x-1}=\frac{2}{x-1}+1=f(x-1)+1$. This is a double transformation consisting of a translation of 1 unit in the x -direction and a translation of 1 unit in the $y$-direction. The asymptotes are now $x=1$ and $y=1$.
e.g. 2: $\quad f(x)=(x-1)^{2}$. The graph of $y=f(x)$ is shown. Sketch on a different axis the graph of $-(x+1)^{2}$.

answer :

$$
-(x+1)^{2}=-\left[(-x-1)^{2}\right]=-f(-x)
$$

This is a double transformation consisting of a reflection in the y-axis $\{f(-x)\}$ followed by a reflection in the x -axis.

exercise 1 :
1.

$$
f(x)=x^{2}
$$

State the transformations that map $f(x)$ onto
(i) $(x-2)^{2}$
(ii) $\quad x^{2}+2 \mathrm{x}+1$
(iii) $x^{2}-2 \mathrm{x}+2$

State the images of $A$ in each case.
2. (a) Show $\{$ by long division if necessary $\}$ that

$$
\frac{2 x-1}{x-1}=2+\frac{1}{x-1}
$$

(b) $\quad g(x)=\frac{1}{x}$.

The figure shows the graph of $y=g(x)$. State the transformations that map $g(x)$ onto
(i) $\frac{1}{x-1}$
(ii) $\frac{2 \mathrm{x}-1}{x-1}$

Hence sketch the graph of $\quad y=\frac{2 \mathrm{x}-1}{x-1}$
showing on your sketch the image of A and the asymptotes. \{also state the equations of the asymptotes \}
(c) A function $f: x \rightarrow \frac{2 \mathrm{x}-1}{x-1}, x>1$. State the range of $f$.
3. (a) (i) Show that $3+2 \mathrm{x}-x^{2}=4-(x-1)^{2}$
(ii) Hence describe the transformations that map $x^{2}$ onto $3+2 \mathrm{x}-x^{2}$.
(b) Sketch the graph of $y=3+2 x-x^{2}$
showing the coordinates of

Sketch the graph of $y=3+2 x-x^{2}$
showing the coordinates of


(i) the point where the graph crosses the $y$-axis.
(ii) the point where $3+2 \mathrm{x}-x^{2}$ is a maximum.
answers to exercise 1 :
1.
(i) $\mathrm{A}_{1}(3,1)$,
(ii) $\mathrm{A}_{2}(0,1)$,
(iii) $\mathrm{A}_{3}(2,2)$
2. (b) (i) A translation through 1 unit in the $x$-direction.
(ii) Translations through 1 unit in both x and y directions.
(c) $\quad f(x)>1$
3. (ii) $x^{2}$ is mapped onto $3+2 x-x^{2}$ by the following transformations: a translation of 1 unit in the $x$-direction, a reflection in the $x$-axis, a translation of 4 units in the $y$-direction.

TEST :

1. $\quad f(x)=(x-2)^{2}-1$
(a) (i) State the axis of symmetry of the graph of $y=f(x)$.
(ii) State the minimum value of $f(x)$.
(iii) Evaluate $f(0), f(1), f(3)$ and $f(4)$.,
(b) (i) Sketch the graph of $y=|f(x)|$ for $0 \leq x \leq 4$
(ii) Use your graph to determine the values of $x$ for which $|f(x)|=1$.
2. 

$$
g(x)=\cos x
$$

The diagram shows the graph of

$$
y=g(x) \text { for } 0 \leq x \leq \pi
$$

Sketch the graph of

$$
y=-\cos \left(x-\frac{\pi}{2}\right)
$$

and show the coordinates of the images the points $A$ and $B$ on your graph.

3.

The diagram shows the graph of $y=f(x)$, where $f(x)=e^{x}$.
Sketch on separate axes the graphs of
(i) $y=e^{-x}$
(ii) $y=-e^{-x}$
(iii) $y=3-e^{-x} .0 \leq x \leq 4$

In each case show the coordinates of the image of A .


