N.B. **The symbols for the "natural numbers", "integers", "real numbers" are** N , Z **and** R

$N = \{ 1 , 2 , 3 , 4 , . . . \}$

$Z = \{ . . . -2 , -1 , 0 , 1 ,2 . . . \}$ **and**

R **is any number that can be found on the number line.**

The set of fractions { ***rational numbers*** **} is represented by the letter** Q .

e.g. 1 : The function f maps from A{ **domain** } to B { **co-domain** } where $f : x \rightarrow 2x-1$.

A = $\{ x : x<4, x \in N \}$, B = $\{ x : x<6, x \in N \}$.

(a) (i) Draw an 'arrow diagram' to show this relation .

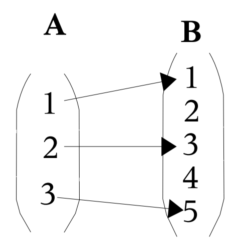
(ii) Describe the correspondence.

(iii) State the **range** of f .

(iv) Give a reason why you would conclude that f is not "**surjective**" { **onto** } .

answer to part (a) :

(i)



(ii) This is a one – to – one correspondence. { f is ***injective*** }

(iii) The '**range**' is represented by the values that $f(x)$ can be. These would be the members of the co-domain that are mapped **onto**. Thus in this instance it is represented by { 1 , 3 , 5 }.

(iv) f is **surjective** only if every member of the co-domain is mapped **onto**. Since $2 \in B$ is not mapped onto, this proves that f is not **surjective**. *

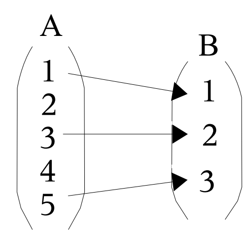
(b) (i) Draw an 'arrow diagram' to show the "inverse" mapping .

(ii) Give a reason why this inverse mapping is not a function.

(iii) Describe a way in which the co-domain of f could be edited so that the inverse mappiing would also be a function. How would you describe the correspondence for f in this instance?

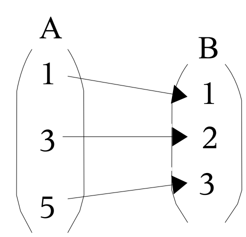
answer to part (b) :

(ii)



(ii) A mapping is regarded as a function only if every element in A has a unique image in B. Since $4 \in A$ has no image in B, this inverse mapping cannot be regarded as a function for all of the elements in A. *

(iii)



(iii) If the co-domain of f were limited to the set { 1, 3 , 5 }, then f would be **bijective,** and the inverse mapping would be a function in this case.

* **This is an example of proof by counter example**

N.B: only one – to - one and many – to – one correspondences are functions

e.g. 2 : The function f maps from A{ **domain** } to B { **co-domain**}where $f\ : x \rightarrow 2x-1$.
A = { $x\ :\ x \in R$ }, B = { $x\ :\ x \in R$ } .

f is **surjective**, since $f(x)$ can be any real number.

Since R is infinite it would be impossible to represent all of f by a mapping diagram.
Thus '**proof by exhaustion**' is not an option for showing that f is **injective**.
We are going to have to resort to an alternative type of proof to arrive at the conclusion that f is one – to – one { **injective** }.
We already saw from example 1 that only many – to – one and one – to – one mappings are functions. We are going to use this fact to our advantage. If we can show that a function is not many – to – one, then the alternative must be accepted if we maintain that the mapping is a function. **
Thus if $f(a_1) = f(a_2)$, $a_1 , a_2 \in A$ $\Rightarrow$ $a_1 = a_2$, we accept that f is injective .
In this example, given $a_1 , a_2 \in A$, $f(a_1) = f(a_2)$ $\Rightarrow$ $2a_1-1 = 2a_2-1$ $\Rightarrow$ $a_1 = a_2$. Therefore f is **injective** . { it can also be shown that f is **bijective** }

To find f^{-1} { the inverse function }, we let $y = f(x)$ $\Rightarrow$ $y = 2x-1$.
The inverse is obtained when we exchange x and y . Thus the inverse relation is
$x = 2y-1$ $\Rightarrow$ $x+1 = 2y$ $\Rightarrow$ $\frac{x+1}{2} = y$. Therefore $f^{-1}\ :\ x \rightarrow \frac{x+1}{2}$.
To find the value of x for which $f(x) = 5$, we say ...
If $f(x) = 5$, then $x = f^{-1}(5) = \frac{5+1}{2} = 3$.

exercise 1 :

1. A function maps from A to B , where $f\ :\ x \rightarrow x^2+5$
A = { $x\ :\ -2 \leq x \leq 2$, $x \in Z$ } and B = { $x\ :\ 0 \leq x \leq 9$ $x \in Z$ }

(a) (i) Draw an 'arrow diagram' to show this relation . (ii) Describe the correspondence.
(iii) State the **range** of f . (iv) Is f **surjective** ? { ***give a reason for your answer*** }

(b) (i) Show the inverse mapping by means of an arrow diagram.
(ii) Explain why the inverse mapping is not a function for all the elements in A.

2. $f\ :\ x \rightarrow \frac{4x}{2x-1}$, $x \in R$

(a) (i) Find the value of x for which f is not defined.
(ii) Find f^{-1} and state the value of x for which f^{-1} is undefined .
(iii) Given that $f(x) = 6$, find x .

(b) Show that $f\ f(x) = \frac{16x}{6x+1}$, $x \neq -\frac{1}{6}$.

answers to the exercise :

1. (ii) { 5 , 6 , 9 }.

2. (i) $\frac{1}{2}$, (ii) $f^{-1} : x \rightarrow \frac{x}{2x-4}$, (iii) $\frac{3}{4}$.

**	**This is an example of proof by contradiction**

TEST1. :

1. The function f is defined on R by $f :\rightarrow 2x+3$

(a) Show that f one – to – one. **[2]**

(b) find the values of $x \in R$ such that $f\ f(x) = 13$. **[2]**

2. The functions f and g are defined on R by

$f :\rightarrow 3x+6$, $g :\rightarrow x+7$.

Solve the equation $f\langle g(2x+1)\rangle = 30$. **[2]**

3. The function f is defined on N by $f :\rightarrow 2x-1$.

By considering $f(x) = 2$, or otherwise, prove that f is not surjective. **[2]**

Find f^{-1} and state two values in the co-domain of f for which f^{-1} is defined. **[4]**

State the range of f . **[2]**

TOTAL MARK = 14

C W L .