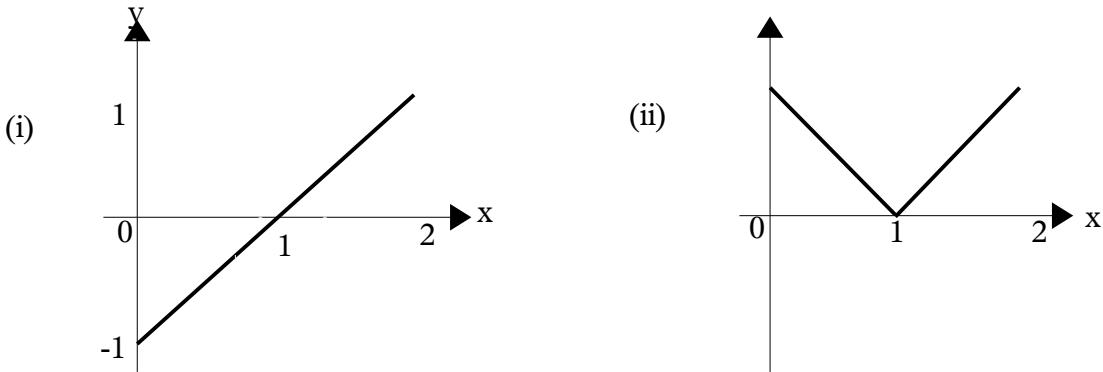


N.B. $|f(x)|$ represents the magnitude of $f(x)$.

e.g.1 $f(x) = x - 1$. On different axes sketch the graphs of (i) $y = f(x)$ and $y = |f(x)|$.

answer:



N.B. The modulus of $|a|$ can be represented by $+a$, if $a \geq 0$ or by $-a$, if $a < 0$.

The modulus of a can also be represented by the positive square root of a^2 .
So that $(|a|)^2$ can be replaced by a^2

e.g. 2 : Find the real values of x for which $|x-2| < 3$

method 1 : If $x-2 \geq 0$ * we could say that $x-2 < 3 \Rightarrow x < 5 \Rightarrow 2 \leq x < 5$
Or if $x-2 < 0$ *, $-(x-2) < 3 \Rightarrow -x+2 < 3 \Rightarrow x > -1 \Rightarrow -1 < x < 2$
Thus we get $-1 < x < 5$.

method 2: If $|x-2| < 3$, we could say that $(x-2)^2 < 3^2 \Rightarrow x^2 - 4x + 4 < 9$.
 $x^2 - 4x - 5 < 0 \Rightarrow (x+1)(x-5) < 0$
From the number line $\Rightarrow x+1 > 0$ and $x-5 < 0 \Rightarrow -1 < x < 5$.

N.B. In general the method that we adopt will determine the number of steps required.

e.g. 3 : Find the real values of x for which $\left| \frac{2x}{x-3} \right| < 1$

answer : If $\left| \frac{2x}{x-3} \right| < 1 \Rightarrow \left(\frac{2x}{x-3} \right)^2 < 1^2 \Rightarrow \left(\frac{2x}{x-3} \right)^2 - 1^2 < 0$.
 $\frac{4x^2 - 1(x^2 - 6x + 9)}{(x-3)^2} < 0 \Rightarrow 4x^2 - x^2 + 6x - 9 < 0$, since $(x-3)^2 > 0$.
 $3x^2 + 6x - 9 < 0 \Rightarrow x^2 + 2x - 3 < 0 \Rightarrow (x+3)(x-1) < 0$.

From the number line, we get $x+3 > 0$ and $x-1 < 0 \Rightarrow -3 < x < 1$.

e.g. 4 : Find the real values of x for which $x^2 - 4|x| + 3 < 0$

method 1 : If $x \geq 0$, $x^2 - 4x + 3 < 0 \Rightarrow (x-1)(x-3) < 0 \Rightarrow x-1 > 0$ and $x-3 < 0$.
 $\Rightarrow 1 < x < 3$.

If $x < 0$, $x^2 + 4x + 3 < 0 \Rightarrow (x+1)(x+3) < 0 \Rightarrow x+1 < 0$ and $x+3 > 0$.
 $\Rightarrow -3 < x < -1$.

method 2 : $x^2 - 4|x| + 3 < 0 \Rightarrow x^2 + 3 < 4|x|$
 $(x^2 + 3)^2 < 16x^2 \Rightarrow x^4 + 6x^2 + 9 - 16x^2 < 0 \Rightarrow x^4 - 10x^2 + 9 < 0$.

This can be written as $(x^2 - 1)(x^2 - 9) < 0$.

Either $x^2 - 1 > 0$ and $x^2 - 9 < 0$.

For $x^2 - 1 > 0$, we get $(x-1)(x+1) > 0$.

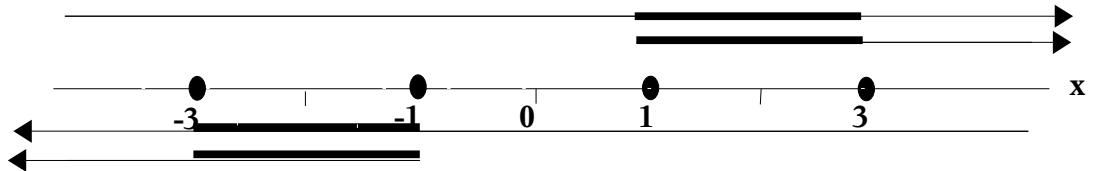
This implies that $x-1 < 0$ and $x+1 < 0 \Rightarrow x < -1$.

or $x-1 > 0$ and $x+1 > 0 \Rightarrow x > 1$.

For $x^2 - 9 < 0$, we get $(x-3)(x+3) < 0$.

This implies that $x-3 < 0$ and $x+3 > 0 \Rightarrow -3 < x < 3$.

These statements agree where $1 < x < 3$ or $-3 < x < -1$, as illustrated below.



exercise 1 :

1. Find the real values of x for which $|2x-5| = 9$

2. Find the real values of x for which $|x+1| < 4$

3. Find the real values of x for which $\left|\frac{2x-1}{x+1}\right| < 1$

4. Find the real values of x for which $x^2 - |x| < 2$ {use any method}.

answers to exercise 1 :

1. $x=7$ and $x=-2$.

2. $-5 < x < 3$.

3. $0 < x < 2$.

4. $-2 < x < 2$.

* N.B. $x-2 \geq 0 \Rightarrow x \geq 2$

* N.B. $x-2 < 0 \Rightarrow x < 2$

TEST :

1. $g(x) = x^2 - 4x + 3$

Sketch the graph of $y = g(x)$ for $0 \leq x \leq 4$. [4]

2. Find the real values of x for which $|x| > 2$ [2]

3. Find the real values of x for which $|2x-1| = |x|$ [4]

4. Find the real values of x for which $|x+3| < 4$ [4]

5. Find the real values of x for which $\left| \frac{x-1}{x-3} \right| < 1$ [6]

TOTAL MARK = 20

C W L.