**e.g. 1 :** The number "9" can be written as $3^2$ , $9^1$ , $81^{\frac{1}{2}}$ , $\approx 10^{0.9542}$ .

Therefore $\log_3 9 = 2$ , $\log_9 9 = 1$ , $\log_{81} 9 = \frac{1}{2}$ and $\log 9 \approx 0.9542$

In general $A = a^x \Leftrightarrow \log_a A = x$ *

**e.g. 2 :** $\log_5 25^x = \log_5 (5^2)^x = \log_5 5^{2x} = 2x = x \times 2 = x \log_5 25$ .

In general $\log_a A^x$ can be written as $x \log_a A$ *

**e.g. 3 :** Solve $25^x = 125$

answer:

$$25^x = 125 \Rightarrow (5^2)^x = 5^3 \Rightarrow 5^{2x} = 5^3 \Rightarrow 2x = 3 \Rightarrow x = \frac{3}{2}$$

alternative : $$25^x = 125 \Rightarrow x \log_5 25 = \log_5 125 \Rightarrow x = \frac{\log_5 125}{\log_5 25} = \frac{3}{2}$$

alternative: $$25^x = 125 \Rightarrow x \log 25 = \log 125 \Rightarrow x = \frac{\log 125}{\log 25} = 1.5$$ ***

**e.g. 4 :** Solve the equation $9^x - 10 = 3^{x+2}$ giving your answer correct to 3 significant figures.

answer:

$9^x - 10 = 3^{x+2} \Rightarrow (3^2)^x - 10 = 3^2 \times 3^x$ .

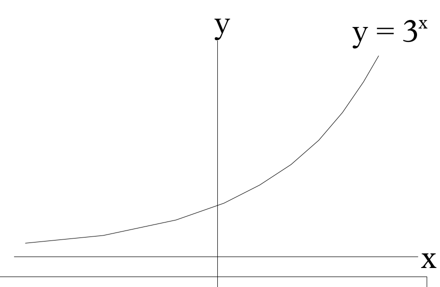
$(3^x)^2 - 9(3^x) - 10 = 0$ .

$(3^x + 1)(3^x - 10) = 0$

Either $3^x = -1 \Rightarrow$ no solution, since $3^x > 0$

Or $3^x = 10 \Rightarrow x \log 3 = \log 10$ .

Therefore $x = \frac{\log 10}{\log 3} \approx 2.10$




note that $3^x$ is always greater than zero

**exercise 1 :**

1 Write numbers to represent each of the following;

(i) $\log_2 32$ , (ii) $\log_2 \frac{1}{64}$ , (iii) $\log_4 \frac{1}{64}$ ,

(iv) $\log_3 \frac{1}{27}$ , (v) $\log 4.13$ { ***giving this answer correct to 4 decimal places*** }

2 Write logarithms of each of the following with base 16 , base 4 , base 2 , and base 10
{***The logarithms in base 10 should be written correct to 4 decimal places.***}

(a) 256 (b) $\frac{1}{16}$ (c) 4 (d) 1

**{use** $\log(A/B) = \log A - \log B$ **}**

3 Write each of the following as single logarithms . **{ use** $\log(AB) = \log A + \log B$ **}**

(i) $x \log_3 a$ , (ii) $\log_a 5 + \log_a 12$ , (iii) $\log_a 48 - \log_a 16$

(iv) $4 \log_a 2 + \frac{1}{2} \log_a 9$ , (v) $\frac{3}{4} \log_a 16 - \frac{2}{3} \log_a 8$

4 (a) Solve the following equations

(i) $16^x = 128$ , (ii) $9^x = 27$ , (iii) $1000^x = 100000$ ,

(iv) $8^x = 4$ , (v) $27^x = 9$ , (vi) $1000^x = 10$ ,

(vii) $4^x = \frac{1}{8}$ , (viii) $3^x = \frac{1}{81}$ , (ix) $100^x = \frac{1}{10}$ ,

(b) Solve the following equations giving your answers correct to 3 significant figures.

(i) $3^x = 14$ , (ii) $7^{x-1} = 2$ , (iii) $(\frac{1}{4})^x > 4$ (iv) $0.2^x < 0.5$ .

5 (a) Solve the following equations

(i) $8^{x-1} = 16$ , (ii) $9^{x-3} = 27$ , (iii) $100^{x+1} = 100000$ ,

(b) Solve the following equations giving your answers correct to 3 significant figures.

(i) $3^{x-2} = 8$ , (ii) $8^{x+1} = 100$ , (iii) $0.8^{x-1} > 0.5$ .

6 (a) Solve the following equations

(i) $3^{2x} + 9 = 10\,(3^x)$ , (ii) $x = (\sqrt{x}) + 6$ , (iii) $2^{2x} = 2^{x+2} + 32$

(b) Solve the equation $4^x = 2^{x+3} + 9$ giving your answer correct to 3 significant figures.

**e.g. 5 :**

(a) Change $\log_x 16$ to a logarithm in base 2 .

(b) Hence solve for $x$ , $\log_2 x + \log_x 16 = 5$ .

answer :

(a) $\log_x 16 = \dfrac{\log_2 16}{\log_2 x} = \dfrac{4}{\log_2 x}$ .

(b) If $\log_2 x + \log_x 16 = 5$ $\Rightarrow$ $\log_2 x + \dfrac{4}{\log_2 x} = 5$

$\log_2 x - 5 + \dfrac{4}{\log_2 x} = 0$ $\Rightarrow$ $\dfrac{(\log_2 x)^2 - 5\log_2 x + 4}{\log_2 x} = 0$

$(\log_2 x)^2 - 5\log_2 x + 4 = 0$ $\Rightarrow$ $(\log_2 x - 4)(\log_2 x - 1) = 0$

Either $\log_2 x = 4$ $\Rightarrow$ $x = 2$ , Or $\log_2 x = 1$ $\Rightarrow$ $x = 0$ .

exercise 2 :

1. (a) Write $\log_x 81$ as a logarithm in base 3 .

   (b) Hence solve for $x$ , $\log_3 x + \log_x 81 = 4$ .

2. Solve $\log_2 x = \log_x 16$ .

3. Change the following to logarithms with base 2 .

   (i) $\dfrac{1}{\log_x 8}$ , (ii) $\dfrac{1}{\log_x 32}$ , (iii) $\log_x 2$ , (iii) $1 \div \log_x\left(\frac{1}{2}\right)$ .

answers to the questions in the exercise 1:

1 (i) 5 , (ii) -6 , (iii) -3 , (iv) -3 , (v) 0.6160

2 (a) 2 , 4 , 8 , 2.4082 . (b) -1 , -2 , -4 , -1.2041 .
(c) 0.5 , 1 , 2 , 0.6020 . (d) 0 , 0 , 0 , 0

3 (i) $\log_3 a^x$ , (ii) $\log_a 60$ , (iii) $\log_a 3$ , (iv) $\log_a 48$ , (v) $\log_a 2$ .

4 (a) (i) $\frac{7}{4}$ , (ii) $\frac{3}{2}$ , (iii) $\frac{5}{3}$ , (iv) $\frac{2}{3}$ , (v) $\frac{2}{3}$ , (vi) $\frac{1}{3}$ , (vii) $-\frac{3}{2}$ , (viii) $-4$ , (ix) $-\frac{1}{2}$
(b) (i) 2.40 , (ii) 1.36 , (iii) $x < -1$ , (iv) $x > 0.431$ .

5 (a) (i) $\frac{7}{3}$ , (ii) $\frac{9}{2}$ , (iii) $\frac{3}{2}$ .
(b) (i) 3.89 , (ii) 1.21 , (iii) $x < 4.11$ .

6 (a) (i) $x=0$ or $x=2$ , (ii) $x=4$ or $x=9$ , (iii) $x=3$ . (b) 3.17 .

answers to exercise 2 :

1. (a) $\dfrac{4}{\log_3 x}$ , (b) $x=9$

2. $x = 4$ , $x = \frac{1}{4}$ .

3. (i) $\frac{1}{3}\log_2 x$ , (ii) $\frac{1}{5}\log_2 x$ , (iii) $\dfrac{1}{\log_2 x}$ , (iv) $-\log_2 x$ or $\log_2\left(\frac{1}{x}\right)$ .

**TEST 1 :**

***If the exact answer cannot be obtained, then write your answer correct to 3 significant figures, unless otherwise instructed.***

**1.** (a) Write $\log A + \log M - \log X$ as a single logarithm . **[ 1 ]**

(b) (i) Write 4 as a logarithm in base 2 **[ 1 ]**

(ii) Write $3 \log_2 A$ as a single logarithm . **[ 1 ]**

(iii) Hence write $4 + 3 \log_2 A$ as a single logarithm. **[ 1 ]**

(c) Given that $\log_3 y = x$ , express $y$ in terms of $x$ . **[ 1 ]**

**2.** (a) Simplify $\left[\frac{8}{27}\right]^{-\frac{2}{3}}$ writing your answer as a mixed number . **[ 3 ]**

(b) Solve for $x$ , $x \in R$ .

(i) $3^{x-2} = 4$ **[ 3 ]**

(ii) $x \log 0.25 > \log 0.5$ . **[ 2 ]**

(c)

(i) Show that $2^{x+1}$ is equivalent to $2\,(2^x)$ **[ 2 ]**

(ii) Hence or otherwise solve for $x$ , $x \in R$ ,

$4^x - 2^{x+1} = 8$ **[ 4 ]**

3. Solve for $x \in R$ , $\log_4 x + 1 = \log_x 16$ . **[ 5 ]**

TOTAL = 24

**C W L**